

Converting between Radical Form and Rational Exponent Form

The diagram illustrates the conversion between radical and rational exponent forms. On the left, a radical expression is shown: a square root symbol with a horizontal line extending to the right, containing a box with the letter 'a' on the left and a box with the letter 'b' on the right. Below the radical symbol is a large, bold 'X' with a blue dashed underline. In the center, there is an equals sign (=) enclosed in a dashed rectangular box with a rotation handle at the top. On the right, a rational exponent expression is shown: a large, bold 'X' with a blue dashed underline, followed by a fraction where the numerator is a box with the letter 'b' and the denominator is a box with the letter 'a'.

$$\sqrt{a \cdot b} = X \cdot \frac{b}{a}$$

<p>Example 1 Convert the following to a rational exponent $\sqrt[5]{x^2 y^3 z^{10}}$</p> $x^{\frac{2}{5}} y^{\frac{3}{5}} z^{\frac{10}{5}}$ $x^{\frac{2}{5}} y^{\frac{3}{5}} z^2$	<p>Example 2 Convert the following to a radical $x^{\frac{1}{2}} y^{\frac{5}{4}}$</p> <p style="text-align: right;"><u>exp</u> Root index</p> $x^{\frac{2}{4}} y^{\frac{5}{4}}$ $\sqrt[4]{x^2 y^5}$
<p>Example 3 Simplify the following $(x^{\frac{2}{3}} y^{\frac{4}{9}})^{\frac{9}{2}}$</p> $x^{\frac{2}{3} \cdot \frac{9}{2}} y^{\frac{4}{9} \cdot \frac{9}{2}}$ $x^6 y^2$	<p>Example 4 Simplify the following $x^{\frac{1}{5}} x^{\frac{2}{3}}$</p> $x^{\frac{1}{5} + \frac{2}{3}} = x^{\frac{3}{15} + \frac{10}{15}}$ $x^{\frac{13}{15}}$

$$\textcircled{4} \quad \frac{1}{(\sqrt[3]{7b})^2} = \frac{1}{(7b)^{\frac{2}{3}}} = (7b)^{\frac{2}{3}}$$

$$\textcircled{9} \quad \left(a^{\frac{1}{2}}\right)^{\frac{5}{3}} \cdot a^{\frac{3}{4}} \\ a^{\frac{5}{6}} \cdot a^{\frac{3}{4}} = a^{\frac{5}{2}} a^{\frac{3}{4}} = a^{\frac{10}{4}} a^{\frac{3}{4}} \\ = a^{\frac{13}{4}}$$

$$\left(\frac{x^{-2} y^{\frac{1}{2}} x^{\frac{7}{4}} y^{\frac{1}{2}} x^2}{x^1} \right)^{-2}$$

$$\frac{7}{4} \cdot \frac{-2}{1} = \frac{-14}{4}$$

$$\frac{x^4 y^{-1} x^{-\frac{14}{4}} y^{-1} x^{-4}}{x^{-2}} = \frac{x^{-\frac{7}{2}} y^{-2}}{x^{-2}}$$

$$x^{-\frac{7}{2} - (-2)} y^{-2} = x^{-\frac{3}{2}} y^{-2} = \frac{1}{x^{\frac{3}{2}} y^2}$$