

## Completing the

Square

$$ax^2 + bx + c = 0$$

WHY?

This method allows us to use the square root method to solve quadratics that cannot be rewritten as  $ax^2 = c$ .

HOW?

Rearrange your equation so it looks like:

$$x^2 + bx = c \quad x^2 + bx + \square = c + \square$$

If  $a \neq 1$ , divide every term by  $a$ .

In the squares, write  $\left(\frac{b}{2}\right)^2$ .

Now, you can rewrite the left side as  $\left(x + \frac{b}{2}\right)^2$  *\* If b is negative  $\left(x - \frac{b}{2}\right)^2$*

Take the square root of each side. Don't forget the  $\pm$ .

Solve for  $x$ .

EXAMPLE:

Solve by completing the square.

$$\text{ex 1) } x^2 + 6x = 5$$

$$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$$

$$x^2 + 6x + 9 = 5 + 9$$

$$(x+3)^2 = 14$$

$$\sqrt{(x+3)^2} = \sqrt{14}$$

$$x+3 = \pm\sqrt{14}$$

$$\begin{array}{r} x+3 = \pm\sqrt{14} \\ -3 \quad \cdot 3 \\ \hline x = -3 \pm \sqrt{14} \end{array}$$

$$\begin{array}{l} x^2 + 6x + 9 \\ \text{mult. 9} \\ \text{Add 6} \\ (x+3)(x+3) \end{array}$$

$$\begin{aligned} \textcircled{\text{ex2}} \quad x^2 + 8x + 9 &= 0 \\ &\quad \quad \quad -9 \quad -9 \\ \hline x^2 + 8x &= -9 \\ \left(\frac{8}{2}\right)^2 &= (4)^2 = 16 \\ x^2 + 8x + 16 &= -9 + 16 \\ (x+4)^2 &= 7 \\ \sqrt{(x+4)^2} &= \sqrt{7} \\ x+4 &= \pm\sqrt{7} \\ -4 \quad -4 \\ \hline x &= -4 \pm \sqrt{7} \end{aligned}$$

$$\text{ex 3 } \frac{2x^2}{2} + \frac{24x}{2} = \frac{26}{2}$$

$$x^2 + 12x = 13$$

$$\left(\frac{12}{2}\right)^2 = (6)^2 = 36$$

$$x^2 + 12x + 36 = 13 + 36$$

$$(x+6)^2 = 49$$

$$\sqrt{(x+6)^2} = \sqrt{49}$$

$$x+6 = \pm 7$$

$$\begin{array}{r} x+6=7 \\ -6 \quad -6 \end{array}$$

$$\begin{array}{r} x+6=-7 \\ -6 \quad -6 \end{array}$$

$x=1$	$x=-13$
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$$\text{ex 4. } x^2 - 7x = 8$$

$$\left(\frac{-7}{2}\right)^2 = (-3.5)^2 = 12.25$$

$$x^2 - 7x + 12.25 = 8 + 12.25$$

$$(x - 3.5)^2 = 20.25$$

$$x - 3.5 = \pm \sqrt{20.25}$$

$$+3.5 \quad +3.5$$

$$x = 3.5 \pm \sqrt{20.25}$$

$$3.5 \pm 4.5$$

$$3.5 + 4.5$$

$$x = 8$$

$$3.5 - 4.5$$

$$x = -1$$

$$x^2 + 10x + \underline{\hspace{2cm}}$$
$$\left(\frac{10}{2}\right)^2 = 25$$
$$(x+5)^2$$

