Use Synthetic Division

Divide.

1) $(x^4 + 10x^3 + 14x^2 - 35x + 22) \div (x + 4)$

$$x+4=0$$
$$\underline{-4 \quad -4}$$
$$x=-4$$

$$\begin{array}{r|rrrrr} -4 & 1 & 10 & 14 & -35 & 22 \\ & \downarrow & -4 & -24 & 40 & -20 \\ \hline & 1 & 6 & -10 & 5 & \boxed{2} \end{array}$$

$$1x^3 + 6x^2 - 10x + 5 + 2/x+4$$

2) $(x^3 - 4x^2 - 13x + 1) \div (x + 1)$

$$\begin{array}{r|rrrr} -1 & 1 & -4 & -13 & 1 \\ & \downarrow & -1 & 5 & 8 \\ \hline & 1 & -5 & -8 & \boxed{9} \end{array}$$

$$x^2 - 5x - 8 + \frac{9}{x+1}$$

Remainder Theorem and Factor Theorem

What is the difference between a factor and a solution?

Solution Factor

$x=$ $(\ x\ )$

What is the difference between a solution and a root? Nothing

Given the factor find the solution:

1. (x – 7) $x-7=0$ $x=7$
2. (3x+1) $3x+1=0$ $x=-\frac{1}{3}$

Given the root find the factor:

3. x = 4 $-4\ -4$ $(x-4)$
4. x = ½

$$x=\frac{1}{2}$$

$$x-\frac{1}{2}=0$$

$$\cdot 2x=\frac{1}{2}\cdot 2$$

$$2x=1$$

$$-1\quad -1$$

$$2x-1$$

there's a $0x^3$.

$$\begin{array}{r|rrrrr} 5 & 3 & 0 & -5 & 1 & -9 \\ & & 15 & 75 & 350 & 1755 \\ \hline & 3 & 15 & 70 & 351 & 1746 \end{array}$$

The remainder is 1746, so $P(5)=1746$.

Let's Try:

5. Use synthetic division to find the value of $P(-3)$ if $P(x) = x^4 - 5x^2 + x + 10$.

$0x^3$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -5 & 1 & 10 \\ & \downarrow & -3 & 9 & -12 & 33 \\ \hline & 1 & -3 & 4 & -11 & \boxed{43} \end{array}$$

$P(-3) = 43$

6. Use synthetic division to find the value of $P(2)$ if $P(x) = x^3 - 12x^2 + 8x + 9$.

$$\begin{array}{r|rrrr} 2 & 1 & -12 & 8 & 9 \\ & \downarrow & 2 & -20 & -24 \\ \hline & 1 & -10 & -12 & \boxed{-15} \end{array}$$

$P(2) = -15$

* if the remainder is zero the value is a solution.

Factor Theorem:

If a remainder is **zero**, the number outside the box in synthetic division is a zero of the function; therefore, the related factor is a factor of the polynomial.

Example: Given $f(x) = 2x^3 + 11x^2 + 18x + 9$, is $(x+3)$ a factor of the polynomial?

$x+3=0$

Because they give you a factor, you must set equal to zero and solve for the solution. Put -3 outside the box, and the coefficients of the polynomial inside the box.

$$\begin{array}{r|rrrr} \boxed{-3} & 2 & 11 & 18 & 9 \\ & & -6 & -15 & -9 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

The remainder is 0, so x = -3 is a zero of the function. That means (x + 3) is a factor of the polynomial

Let's try:

7. Given $f(x) = 2x^3 + 11x^2 + 18x + 9$, is $(x - 1)$ a factor of the polynomial?

$x-1=0$
$+1 \quad +1$
$x=1$

$$\begin{array}{r|rrrr} 1 & 2 & 11 & 18 & 9 \\ & & 2 & 13 & 31 \\ \hline & 2 & 13 & 31 & \boxed{40} \end{array}$$

No, x-1 is Not a factor

8. Given $f(x) = x^4 + 12x^2 + 18x + 9$, is $(x + 3)$ a factor of the polynomial?

$x+3=0$
$x=-3$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 12 & 18 & 9 \\ & & -3 & 9 & -63 & 135 \\ \hline & 1 & -3 & 21 & -45 & \boxed{144} \end{array}$$

No, x+3 is not a factor