Which of the following is a zero?

a. x =2 b. x+1 c. x-3

d. x=-9 e. x=0 f. x

Answers to Factor and Remainder Theorem

1) Yes	2) No	3) Yes	4) No
5) Yes	6) No	7) 2	8) 0 yes
9) 0 yes	10) −7	11) −10	12) 0 yes

12

3	2	-1	-16	-2	15
	↓	6	15	-3	-15
	2	5	-1	-5	$\boxed{0}$

7

2	1	4	-9	-3	-4
	↓	2	12	6	6
	1	6	3	3	$\boxed{2}$

The Rational Zero Theorem:

WHY IT IS IMPORTANT: <u>Narrows the search for rational zeros to a finite list.</u>

- If $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients $(a_n \neq 0)$ and $\dfrac{p}{q}$ is a rational zero (in lowest terms) of p, then

p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.

EX 1: Find the roots of $x^3 + 6x^2 + 10x + 3 = 0$.

HINT: Apply the **Rational Root Theorem** to find the **possible** rational roots!

What is p? 3 : 1, 3 $\frac{1}{1}$ $\frac{3}{1}$

constant

What is q? 1 : 1

L.C.

$\boxed{\text{Possible list: } \pm 1, \pm 3}$

EX 2: Find the possible rational roots of $3x^3 + 5x^2 + 7x + 2 = 0$.

What is p? $\underline{\ 2\ }$: 1, 2 $\quad \frac{1}{1} \quad \frac{2}{1} \quad \frac{1}{3} \quad \frac{2}{3}$
constant
What is q? $\underline{\ 3\ }$: 1, 3
L.C.

Possible List: $\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

EX 3: Find the possible rational roots of $2x^3 + 5x^2 - 6x - 8 = 0$.

What is p? 8 : 1, 2, 4, 8 $\frac{1}{1}$ $\frac{2}{1}$ $\frac{4}{1}$ $\frac{8}{1}$

What is q? 2 : 1, 2 $\frac{1}{2}$ $\frac{2}{2}$ $\frac{4}{2}$ $\frac{8}{2}$

$\boxed{\text{Possible list: } \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}}$

Steps for Finding the Zeros of a Polynomial Function with Integer Coefficients:

1) Gather General Information.
   - Determine the **degree** ***n*** of the polynomial function.
   - The **number of distinct zeros** of the polynomial function is **at MOST** ***n*****.**
2) Check rational zeros.
   - Apply the **Rational Zero Theorem** to list rational numbers that are ***possible*** **zeros**.
   - Use **synthetic division** to test the numbers in the list.
3) Work with the **reduced/depressed** polynomial.
   - Each time a zero is found, obtain the reduced/depressed polynomial.
   - Work to get a **reduced polynomial of degree 2.**
   - Then, find its zeros by ***factoring*** or by applying the ***quadratic formula***.

EX 3: Find the zeros of $f(x) = x^3 - 7x^2 + 16x - 12$.

At most 3 zeros.

Rational Root Theorem – Possible rational zeros:

12 : 1, 2, 3, 4, 6, 12

1 : 1

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Find the roots:

$$\begin{array}{r|rrrr} 2 & 1 & -7 & 16 & -12 \\ & \downarrow & 2 & -10 & 12 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$x^2 - 5x + 6$

$(x-2)(x-3) = 0$

$x - 2 = 0 \qquad x - 3 = 0$

$x = 2 \qquad x = 3$

$$\boxed{\begin{array}{l} x = 2 \\ x = 2 \\ x = 3 \end{array}}$$

EX 4: Find the zeros of $g(x) = 3x^4 + 23x^3 + 56x^2 + 52x + 16$

At most 4 zeros.

Rational Root Theorem – Possible rational zeros:

$\frac{16}{3}$: $\frac{1, 2, 4, 8, 16}{1, 3}$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}$

Find the roots:

-1	3	23	56	52	16
	↓	-3	-20	-36	-16
	3	20	36	16	$\boxed{0}$

$3x^3 + 20x^2 + 36x + 16$

-2	3	20	36	16
	↓	-6	-28	-16
	3	14	8	$\boxed{0}$

$$\boxed{\begin{array}{l} x = -\frac{2}{3} \\ x = -4 \\ x = -2 \\ x = -1 \end{array}}$$

$3x^2 + 14x + 8$

Factor

ac: 24
~~Add~~: 14

$3x^2 + 2x + 12x + 8$

$x(3x+2) + 4(3x+2)$

$(3x+2)(x+4)$

$3x + 2 = 0$ $\quad x + 4 = 0$

$-2 \;\; -2$ $\quad x = -4$

$3x = -2$

$x = -2/3$

Quad

$$\frac{-14 \pm \sqrt{(14)^2 - 4(3)(8)}}{2(3)}$$

$$\frac{-14 \pm \sqrt{100}}{6} = \frac{-14 \pm 10}{6}$$

$$\frac{-14+10}{6} \qquad \frac{-14-10}{6}$$

$$\frac{-4}{6} = -\frac{2}{3} \qquad \frac{-24}{6} = -4$$