Algebra 2
Unit 3 Polynomial Review

Name Key
Date _____ Block_____

Show work on ALL questions! You may need additional paper for some problems.

1. Use the Remainder Theorem to evaluate the function
$f(x) = -x^3 - 2x^2 + x; x = -3$

$f(-3) = 6$

$-3 \mid -1 \quad -2 \quad 1 \quad 0$
$\quad\quad\downarrow \quad 3 \quad -3 \quad 6$
$\quad\quad -1 \quad 1 \quad -2 \quad \boxed{6}$

2. Use the Remainder Theorem to determine whether x = 3 is a zero of the polynomial function
$f(x) = x^3 + x^2 - 16x + 20$. Show work and EXPLAIN.

No, b/c $f(3) \neq 0$

$3 \mid 1 \quad 1 \quad -16 \quad 20$
$\quad\quad\downarrow \quad 3 \quad 12 \quad -12$
$\quad\quad 1 \quad 4 \quad -4 \quad \boxed{8}$

3. Use the Factor Theorem to determine whether (x + 4) is a factor of the polynomial function
$f(x) = x^3 + x^2 - 5x + 28$. Show work and EXPLAIN.

Yes b/c $f(-4) = 0$

$x + 4 = 0$
$-4 \quad -4$
$x = -4$

$-4 \mid 1 \quad 1 \quad -5 \quad 28$
$\quad\quad\downarrow \quad -4 \quad 12 \quad -20$
$\quad\quad 1 \quad -3 \quad 7 \quad \boxed{0}$

4. Factor the polynomial $f(x) = x^3 - x^2 - 17x - 15$ completely, given that $x + 3$ is a factor.

$(x-5)(x+1)(x+3)$

$x + 3 = 0$
$-3 \quad -3$
$x = -3$

$-3 \mid 1 \quad -1 \quad -17 \quad -15$
$\quad\quad\downarrow \quad -3 \quad 12 \quad 15$
$\quad\quad 1 \quad -4 \quad -5 \quad \boxed{0}$

$x^2 - 4x - 5$
$(x-5)(x+1)$

$\frac{5}{1 \cdot 5}$

5. Find d so that $x+3$ is a factor of the polynomial $f(x)=2x^3+7x^2+17x+d$

$-3 \mid 2 \;\; 7 \;\; 17 \;\; d$
$\quad\quad -6 \;\; -3 \;\; -42$
$\quad 2 \;\; 1 \;\; 14 \;\; \boxed{0}$

$x+3=0$
$-3 \;\; -3$
$x=-3$

$d-42=0$
$d=42$

6. Find all the zeros of the polynomial $f(x)=2x^3+9x^2-11x-30$, given that $f(2)=0$

$2 \mid 2 \;\; 9 \;\; -11 \;\; -30$
$\quad\quad 4 \;\; 26 \;\; 30$
$\quad 2 \;\; 13 \;\; 15 \;\; \boxed{0}$

$2x^2+13x+15$

$x=\dfrac{-13\pm\sqrt{(13)^2-4(2)(15)}}{2(2)}$

$x=\dfrac{-13\pm\sqrt{49}}{4}$

$x=\dfrac{-13+7}{4} \quad x=\dfrac{-13-7}{4}$

$x=\dfrac{-6}{4} \quad x=\dfrac{-20}{4}$

$x=-5 \quad x=\dfrac{-3}{2} \quad x=2$

7. List all possible rational zeros of the polynomial function, do NOT find the actual zeros.

$f(x)=2x^4+5x^3-6x^2-7x+6$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

8. Find all zeros of the function.

a.) $f(x) = x^3 - x^2 - 17x - 15$.

$x = 5 \quad x = -3 \quad x = -1$

work

$-1 \mid 1 \quad -1 \quad -17 \quad -15$

$\quad\quad\downarrow \quad -1 \quad 2 \quad 15$

$\quad\quad 1 \quad -2 \quad -15 \quad \boxed{0}$

$x^2 - 2x - 15 \qquad 15: \ 1\ 15,\ 3\ 5$

$(x+3)(x-5)$

$x+3=0 \quad x-5=0$

$-3\ -3 \quad +5\ +5$

$x=-3 \quad x=5$

b.) $g(x) = x^3 - 6x^2 + 9x - 2$

$x = 2 \quad x = 2 \pm \sqrt{3}$

work

$2 \mid 1 \quad -6 \quad 9 \quad -2$

$\quad\quad\downarrow \quad 2 \quad -8 \quad 2$

$\quad\quad 1 \quad -4 \quad 1 \quad \boxed{0}$

$x^2 - 4x + 1 = 0$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$

$x = \dfrac{4 \pm \sqrt{12}}{2}$

$x = \dfrac{4 \pm 2\sqrt{3}}{2} \quad x = 2 \pm \sqrt{3}$

9. How many solutions does the equation $7k^3 + 5k^2 - k + 4 = 0$ have? Explain.

3 b/c deg = 3

10. If $\sqrt{5}$ is a zero of a polynomial function, what must also be a zero?

$X = -\sqrt{5}$

11. If $-2 - 3i$ is a zero of a polynomial function, what must also be a zero?

$X = -2 + 3i$

12. Write the simplest polynomial function in standard form with zeros of -3 (multiplicity 2), and 0.

$X = -3 \quad X = -3 \quad X = 0$

$(x+3)(x+3)(x)$

$f(x) = x^3 + 6x^2 + 9x$

$(x+3)(x+3)$

$x^2 + 3x + 3x + 9$

$x^2 + 6x + 9$

$x(x^2 + 6x + 9)$

$x^3 + 6x^2 + 9x$

13. Write the simplest polynomial function in standard form with zeros of 5 and $2i$.

$f(x) = x^3 - 5x^2 + 4x - 20$

work

$x = 5 \quad x = 2i \quad x = -2i$

$(x-5)(x-2i)(x+2i)$

$(x-5)(x^2+2ix-2ix-4i^2)$

$(x-5)(x^2-4(-1))$

$(x-5)(x^2+4)$

$f(x) = x^3 - 5x + 4x - 20$

14. Given the function $f(x) = x^5(x-4)^4(x+1)^3$, state the zeros of the function including multiplicity.

$x = 0$ mult. 5

$x = 4$ mult. 4

$x = -1$ mult. 3

15. The storage space in a moving truck is shaped like a rectangular prism. It has a volume of 48 cubic meters. The width is 4 meters less than the length, and the height is 2 meter less than the length. Find the dimensions (length, width, height) of the storage space. ***You must write and solve a polynomial equation to receive credit.***

$V=48$ L $W=L-4$ $h=L-2$

$L(L-4)(L-2)=48$

$L(L^2-2L-4L+8)=48$

$L(L^2-6L+8)=48$

$L^3-6L^2+8L=48$

$V=L\cdot W\cdot h$

$L^3-6L^2+8L-48=0$

$L^2(L-6)+8(L-6)=0$

$(L^2+8)(L-6)=0$

$L^2+8=0$ $L-6=0$

$L=\pm 2i\sqrt{2}$ $L=6\checkmark$

Length = 6

Width = 2

Height = 4

16. The storage space in a moving truck is shaped like a rectangular prism. It has a volume of 16 cubic meters. The length and height are each 2 meters less than the width. What is the width of the storage space? ***You must write and solve a polynomial equation to receive credit.***

$V=16$ $L=W-2$ $h=W-2$ $W=W$

$(W-2)(W-2)(W)=16$

$(W^2-2W-2W+4)(W)=16$

$(W^2-4W+4)(W)=16$

$(W^3-4W^2+4W)=16$

$W^3-4W^2+4W-16=0$

$W^2(W-4)+4(W-4)=0$

$W^2+4=0$ $W-4=0$

$W^2=-4$ $W=4$

$W=\pm 2i$

True or False (include a brief explanation):

17. A polynomial function will always have exactly one y-intercept. True

18. A polynomial function can only have one x-intercept. False

19. The **domain** of a polynomial function is always $(-\infty, \infty)$. True

20. The **range** of a quadratic polynomial function is always $(-\infty, \infty)$. False

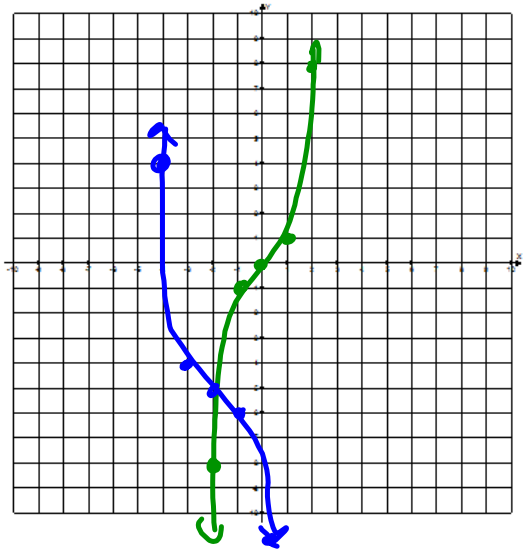
21. Cubic polynomials never have an absolute minimum or maximum. True

22. **Sketch the graph of the given function using the parent graph and transformations as yo**

Complete the table below accordingly (not all may apply): $y = -(x+2)^3 - 5$

Parent Function	Vertical Reflection?	Vertical Stretch? Shrink? What's the scale factor?	Horizontal Shift Right? Left? How many units?	Vertical shift Up? Dow How many units?
x^3	yes	None	Left 2	down 5

Key Points on parent function		Transformed Points to create the graph of your function	
-2	-8	-4	3
-1	-1	-3	-4
0	0	-2	-5
1	1	-1	-6
2	8	0	-13

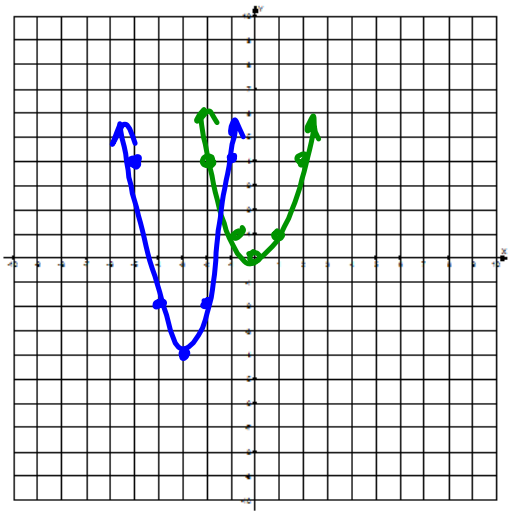


23. **Sketch the graph of the given function using the parent graph and transformations as you**

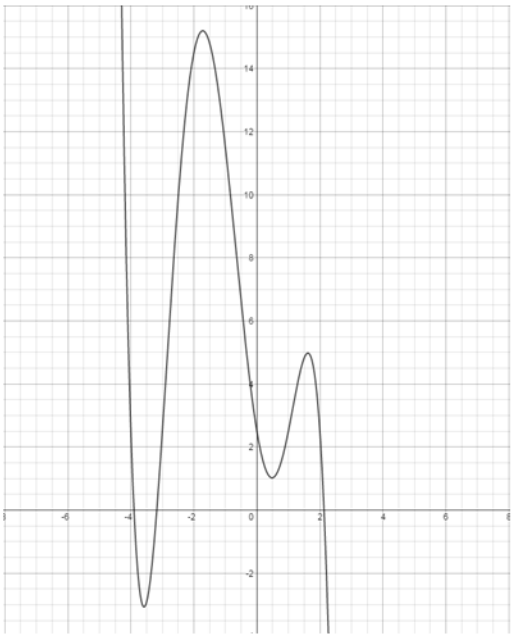
Complete the table below accordingly (not all may apply): $y = 2(x+3)^2 - 4$

Parent Function	Vertical Reflection?	Vertical Stretch? Shrink? What's the scale factor?	Horizontal Shift Right? Left? How many units?	Vertical shift Up? Down How many units?
x^2	none	Stretch by 2	Left 3	down 4

Key Points on parent function		Transformed Points to create the graph of your function	
-2	4	-5	4
-1	1	-4	-2
0	0	-3	-4
1	1	-2	-2
2	4	-1	4



Explain your reasoning for each answer.



24. Is the degree of the function odd or even?

odd b/c ends are opposite directions

25. Is the leading coefficient positive or negative?

negative b/c the end go up to down

26. Is the function an odd function, even function, or neither?

Neither there is no Symmetry

27. What must be the least degree of the polynomial?

5 b/c there are 4 turns

28. Complete the information and sketch the graph of the function.

$f(x) = -(x+2)^2(x-1)^2$

Zeros: $x+2=0$ $x-1=0$

$x=-2$ $x=1$

Multiplicity:

-2 multiplicity of 2

1 multiplicity of 2

y-intercept:

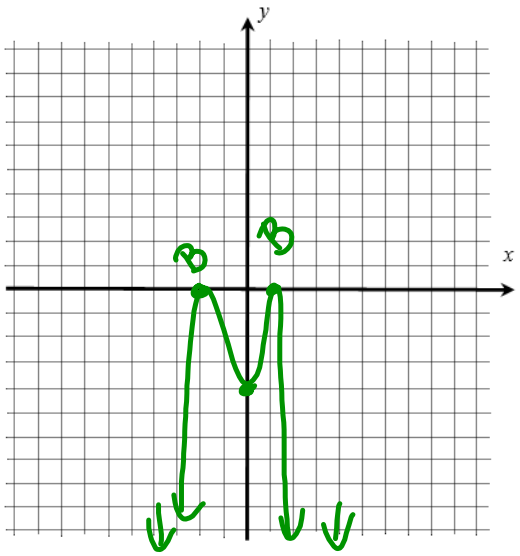
$f(x) = -(0+2)^2(0-1)^2$

$f(x) = -4$ $(0,-4)$

Sketch end behavior based on degree and LC:

Even/neg

↓ ↓



29. Using technology to find intercepts and extrema, then graph the function. Identify the characteristics.

A. y-intercept: (0, 1)

B. zero(s): x = 3.064

C. Relative Minimum(s): (0.239, 0.656)

Relative Maximum(s): (2.095, 7.048)

Absolute Minimum(s): none

Absolute Maximum(s): none

D. Domain: ℝ

E. Range: (−∞, ∞) or ℝ

F. Intervals of increase or decrease:

G. End behavior
$as\ x \to -\infty, f(x) \to \infty$
$as\ x \to +\infty, f(x) \to -\infty$

H. Is this function odd, even or neither?

Neither