Answers to Rational Root Theorem

1) $\pm 1, \pm 2, \pm \frac{1}{2}$

2) $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}, \pm \frac{16}{3}$

3) ± 1

4) $\pm 1, \pm 3, \pm \frac{1}{3}$

5) $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{2}{3}$

6) $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{4}{5}, \pm \frac{1}{10}$

7) Possible rational zeros: $\pm 1, \pm 2, \pm \frac{1}{2}$

Rational zeros: $\left\{\frac{1}{2}, 2, -1\right\}$

8) Possible rational zeros: $\pm 1, \pm \frac{1}{3}$

Rational zeros: $\left\{-\frac{1}{3}, -1 \text{ mult. } 2\right\}$

9) Possible rational zeros: $\pm 1, \pm 2, \pm \frac{1}{2}$

Rational zeros: $\left\{\frac{1}{2}, 2, 1\right\}$

10) Possible rational zeros: $\pm 1, \pm \frac{1}{3}$

Rational zeros: $\left\{-1, 1, -\frac{1}{3}\right\}$

11) Possible rational zeros: $\pm 1, \pm \frac{1}{5}$

Rational zeros: $\left\{-1, \frac{1}{5}, 1\right\}$

12) Possible rational zeros: $\pm 1, \pm \frac{1}{3}$

Rational zeros: $\left\{\frac{1}{3}, -1 \text{ mult. } 2\right\}$

8) $f(x) = 3x^3 + 7x^2 + 5x + 1$

3 answers: $x = -1$ $x = -1$ $x = -\frac{1}{3}$

$p: 1 \to 1$

$q: 3 \to 1, 3$

$\pm 1, \pm \frac{1}{3}$

$$\begin{array}{r|rrrr} -1 & 3 & 7 & 5 & 1 \\ & \downarrow & -3 & -4 & -1 \\ \hline & 3x^2 & 4x & 1 & \boxed{0} \\ & a & b & c & \end{array}$$

mult. ac
$3(1) = 3$
Add $= 4$

$3x^2 + 3x + 1x + 1$

$3x(x+1) + 1(x+1)$

$(x+1)(3x+1)$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(1)}}{2(3)}$$

$$\frac{-4 \pm \sqrt{4}}{6}$$

$\frac{-4+2}{6} = \frac{-2}{6}$ $-\frac{1}{3}$

$\frac{-4-2}{6}$ $\frac{-6}{6} = -1$

The Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra:

If $P(x)$ is a polynomial function of **degree** $n \geq 1$ with complex coefficients, then $P(x)$ has at least one complex zero.

The Linear Factor Theorem:

If $P(x)$ is a polynomial function of degree $n \geq 1$ with leading coefficient $a_n \neq 0$,

then $P(x)$ has exactly n linear factors.

$P(x) = a_n(x - c_1)(x - c_2)(x - c_3)\cdots(x - c_n)$ where c_1, c_2, … , c_n are complex numbers.

(real and/or imaginary)

The Number of Zeros of a Polynomial Function Theorem:

If $P(x)$ is a polynomial function of degree $n \geq 1$, then $P(x)$ has exactly n complex zeros, provided each zero is counted according to its multiplicity.

What does this all mean?

1. you should always have a solution

2. # of factors should = # of solutions

   ex. solutions → factors
   $x=2$
   $x=7$
   $x=-5$
   $(x-2)(x-7)(x+5)$

3. 3 types of solutions

   A. rational real numbers (nice answers)

   B. irrational real numbers (square roots)
   ↳ Always comes in pairs (conjugates)
   $x=-\sqrt{10}$ must have $x=+\sqrt{10}$
   $x=2+\sqrt{7}$ " $x=2-\sqrt{7}$

   C. imaginary numbers
   ↳ Always comes in pairs (conjugates)
   $x=3i$ must have $x=-3i$

EX 1] **Find all zeros** of the polynomial function

$$P(x) = x^4 - 6x^3 + 10x^2 + 2x - 15$$

$p: 15 \rightarrow 1, 3, 5, 15$

$q: 1 \rightarrow 1$

$\pm 1, \pm 3, \pm 5, \pm 15$

$$\begin{array}{r|rrrrr} -3 & 1 & -6 & 10 & 2 & -15 \\ & \downarrow & -3 & 27 & -111 & \\ \hline & 1 & -9 & 37 & -109 & \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 1 & -6 & 10 & 2 & -15 \\ & \downarrow & -1 & 7 & -17 & 15 \\ \hline & 1 & -7 & 17 & -15 & \boxed{0} \end{array}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$\begin{array}{r|rrrr} 3 & 1 & -7 & 17 & -15 \\ & \downarrow & 3 & -12 & 15 \\ \hline & 1 & -4 & 5 & \boxed{0} \end{array}$$

$$\frac{4 \pm \sqrt{-4}}{2}$$

$$\frac{4 \pm 2i}{2}$$

$$x^2 - 4x + 5 = 0$$

$$x = 2 \pm i$$

$$\boxed{x = -1 \quad x = 3 \quad x = 2+i \quad x = 2-i}$$

EX 2] Find all zeros of the polynomial function

$P(x) = x^3 + x^2 - x + 15$

$\pm 1, \pm 3, \pm 5, \pm 15$

$$\begin{array}{r|rrrr} 5 & 1 & 1 & -1 & 15 \\ & \downarrow & 5 & 30 & \\ \hline & 1 & 6 & 29 & \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -1 & 15 \\ & \downarrow & -3 & 6 & -15 \\ \hline & 1 & -2 & 5 & \boxed{0} \end{array}$$

$x^2 - 2x + 5 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{-16}}{2}$$

$$x = \frac{2 \pm 4i}{2}$$

$$\boxed{x = -3 \quad x = 1 + 2i \quad x = 1 - 2i}$$

EX 3 Find all zeros of the polynomial function given that (x+3) is a factor $P(x) = x^4 - 17x^2 + 72$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -17 & 0 & 72 \\ & \downarrow & -3 & 9 & +24 & -72 \\ \hline & 1 & -3 & -8 & 24 & \boxed{0} \end{array}$$

$$x^3 - 3x^2 - 8x + 24$$

$$x^2(x-3) - 8(x-3)$$

$$(x-3)(x^2-8)=0$$

$$x-3=0 \qquad x^2-8=0$$

$$x=3 \qquad \sqrt{x^2}=\sqrt{8}$$

$$x = \pm 2\sqrt{2}$$

$$\boxed{x=3 \quad x=-3 \quad x=2\sqrt{2} \quad x=-2\sqrt{2}}$$