Solving Exponential Equations & Inequalities

*only make base smaller

To solve an exponential equation, create common bases and set the exponents equal to each other and solve.

BASIC EXPONENTIALS: ***To work the following, set the exponents equal to each other and solve.***

1) $7^{3x+8} = 7^{2x-5}$

$3x+8 = 2x-5$
$-2x \quad -2x$
$x+8 = -5$
$-8 \quad -8$
$x = -13$

2) $5^{-2x} = 5^{3x-10}$

$-2x = 3x-10$
$-3x \quad -3x$
$-5x = -10$
$\frac{-5}{} \quad \frac{-5}{}$
$x = 2$

A LITTLE LESS BASIC EXPONENTIALS: *To work these, you will need to rewrite the terms using the same base, then solve.*

3) $2^{2x} = 8^4$

$2^{2x} = (2^3)^4$

$2^{2x} = 2^{12}$

$\frac{2x}{2} = \frac{12}{2}$

$x = 6$

4) $3^{x+3} = 27^2$

$3^{x+3} = (3^3)^2$

$3^{x+3} = 3^6$

$x + 3 = 6$

$-3 \quad -3$

$x = 3$

5) $125^{3x} = 5^{4x+10}$

$(5^3)^{3x} = 5^{4x+10}$

$5^{9x} = 5^{4x+10}$

$9x = 4x + 10$

$-4x \quad -4x$

$\frac{5x}{5} = \frac{10}{5}$

$x = 2$

NOT SO BASIC EXPONENTIALS: ***Rewrite both sides using the same base, then solve for x.***

6) $8^{2x} = 16^{3}$

$(2^3)^{2x} = (2^4)^3$

$2^{6x} = 2^{12}$

$\frac{6x}{6} = \frac{12}{6}$

$x = 2$

7) $25^{x+2} = 625^{2x-10}$

$(25)^{x+2} = (25^2)^{2x-10}$

$25^{x+2} = 25^{4x-20}$

$x+2 = 4x-20$

$-x \quad -x$

$2 = 3x-20$

$+20 \quad +20$

$22 = 3x$

$\frac{22}{3} = \frac{3x}{3}$

$x = \frac{22}{3}$

EXPONENTIAL INEQUALITIES:* ***Remember, when solving inequalities you need to flip the inequality sign when dividing or multiplying by a negative number. You also need to check your solutions to make sure they make sense.***

8) $3^{12} > 9^{2x}$

$3^{12} > (3^2)^{2x}$

$3^{12} > 3^{4x}$

$\frac{12}{4} > \frac{4x}{4}$

$3 > x$

$x < 3$

Check

$3^{12} > 9^{2(2)}$

$3^{12} > 9^4$

$531{,}441 > 6561$

9) $27^{x-2} \leq 81^{x+7}$

$(3^3)^{x-2} \leq (3^4)^{x+7}$

$3^{3x-6} \leq 3^{4x+28}$

$3x-6 \leq 4x+28$

$-4x \quad -4x$

$-1x-6 \leq 28$

$+6 \quad +6$

$\frac{-1x}{-1} \leq \frac{34}{-1}$

$x \geq -34$

Check $-10+7$

$27^{-10-2} \leq 81_{-3}$

$27^{-12} \leq 81$

$6.66 \times 10^{-18} \leq .00000188$

10) $16^{-1} < 64^{-x-2}$

$(4^2)^{-1} < (4^3)^{-x-2}$

$4^{-2} < 4^{-3x-6}$

$-2 < -3x-6$

$+6 \quad +6$

$\frac{4}{-3} < \frac{-3x}{-3}$

$-\frac{4}{3} > x$

$x < -\frac{4}{3}$

Check

$16^{-1} < 64^{-(-2)-2}$

$16^{-1} < 64^0$

$.0625 < 64^0$

$.0625 < 1$