Solve for x

1. $3x - 2=5$

2. $(x - 3)^2 = a$

Notes on Inverse Functions

I. Functions

A. **Relation**

1. A **relation** is any correspondence between a set of input values and output values.
2. The set of all inputs (x) is the **domain** of the relation.
3. The set of all outputs (y or f(x)) is the **range** of the relation.

B. **Function**

1. A **function** is a special type of relation in which for every member of its domain (x) is associated with exactly one member of its range (y or f(x)).
2. In other words, a function is a relationship in which each input value has a unique out value. For every "x" there is one and only one value of "y" associated with that "x".
3. Examples:
   a. If you are working at Suds Car Wash for an hourly wage, the relationship between the numbers of hours you work and the resulting income you earn is a function.
   b. Every holder of a social security card in the United States is assigned a nine-digit social security number.

C. **Functional Notation**

1. Functional notation is often used to represent functions.
2. $f(x)$ is read f of x or the value of the function f at x.
3. Example:
   a. If $f(x) = 3x - 2$, then $f(-3) = 3(-3) - 2$; this equals -11, so $f(-3) = -11$

D. **Composite Functions**

1. Applying one function to the answer of another function is called the **Composition of Functions.**

2. $f * g(x)$ or $f(g(x))$ is read f of g of x or the value of the function f at the value of the function g at x.

   $f \circ g(x)$ or $f(g(x))$

3. Example:

   $f(x) = 2x + 1$ and $g(x) = 4x$; so $f(g(x)) = 2(4x) + 1$, so then $f(-2) = 2(4(-2)) + 1$ which will equal -15.

   $2(4x)+1$
   $8x+1$

   $f(g(-2))$

   So, $f(g(-2)) = -15$

II. **Inverse Functions**

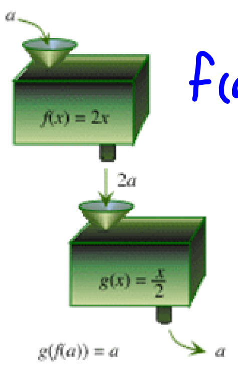
A. Definition:

→ Horizontal line test

1. If $f(x)$ is a one-to-one function with ordered pairs of the form (x, y), its inverse function, $f^{-1}(x)$, is a one-to-one function with ordered pairs of the form (y,x).

2. To determine the inverse of a linear function, you need to derive the function that "undoes" the original function.

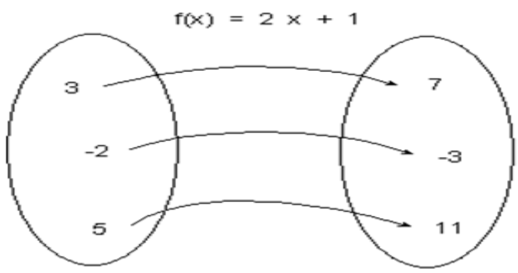
3. Here is a nice way of visualizing what is going on. Since g(f(x)) means "first apply f and then apply g," we can think of this as feeding the output of f into g, and seeing what we get. Here is an illustration of this process.



$f(a) = 2(a)$

B. Overview of a Function and Its' Inverse Function

1. Consider the function f(x) = 2x + 1. This function multiples any number by two and then adds 1. We know how to evaluate f at 3, f(3) = 2*3 + 1 = 7. It helps to think of f as transforming a 3 into a 7, and f transforms a 5 into an 11, etc.



2. Now that we think of f as "acting on" numbers and transforming them, we can define the inverse of f as the function that "undoes" what f did.

3. In other words, the inverse of f needs to take 7 back to 3, and take -3 back to -2, etc. In order to do this the inverse function needs to subtract one to any number and then divide it by 2.

4. Therefore, the Inverse Function would be ☞ g(x) = (x - 1)/2. Then g(7) = 3, g(-3) = -2, and g(11) = 5, so g seems to be undoing what f did, at least for these three values.

5. To prove that g is the inverse of f we must show that this is true for any value of x in the domain of f.

6. In other words, g must take f(x) back to x for all values of x in the domain of f. So, g(f(x)) = x must hold for all x in the domain of f. The way to check this condition is to see that the formula for g(f(x)) simplifies to x.

C. Examples of Functions and their Inverses

Function	Inverse Function
$f(x) = -3x$	g(x) $f^{-1}(x) = \frac{x}{-3}$
$f(x) = 5x + 2$	g(x) $f^{-1}(x) = \frac{x-2}{5}$
$g(x) = x^2$	f(x) ~~$g^{-1}(x)$~~ $= \sqrt{x}$

$f(g(x)) = x$ &

$g(f(x)) = x$

Verify that they are inverses using composition:

$f(g(x)) = -3\left(\frac{x}{-3}\right) = \frac{-3x}{-3} = x$ ✓

$g(f(x)) = \frac{(-3x)}{-3} = \frac{-3x}{-3} = x$ ✓

So f(x) and g(x) are inverses. $g(x) = f^{-1}(x)$ — $^{-1}$ means inverse

$f^{-1}(x)$ read "f inverse of x"

$f(g(x)) = 5\left(\frac{x-2}{5}\right) + 2$

$x - 2 + 2$

x ✓

$g(f(x)) = \frac{(5x+2) - 2}{5} = \frac{5x+2-2}{5}$

$= \frac{5x}{5} = x$ ✓

$f(g(x)) = \sqrt{x^2} = x$ ✓

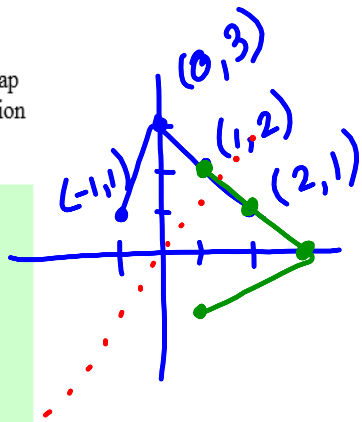
$g(f(x)) = (\sqrt{x})^2$

$= x$ ✓

"So, how do we find inverse functions?"

Consider the following:

1. Swap ordered pairs: If your function is defined as a list of ordered pairs, simply swap the x and y values. Remember, the inverse will be a *function* only if the original function is one-to-one. Examples:



a. Given function f, find the inverse. Is the inverse also a *function*?:
$f(x) = \{(3,4),(1,-2),(5,-1),(0,2)\}$

Answer:
Function f is a one-to-one function since the x and y values are used only once. The inverse is
$f^{-1}(x) = \{(4,3),(-2,1),(-1,5),(2,0)\}$
Since function f is a one-to-one function, the inverse is also a function.

b. Determine the inverse of this function. Is the inverse also a *function*?

x	1	-2	-1	0	2	3	4	-3
$f(x)$	2	0	3	-1	1	-2	5	1

Answser: Swap the x and y variables to create the inverse. Since function f was **not** a one-to-one function (the y value of 1 was used twice), the inverse will **NOT** be a function (because the x value of 1 now gets mapped to two separate y values which is not possible for functions).

x	2	0	3	-1	1	-2	5	1
$f^{-1}(x)$	1	-2	-1	0	2	3	4	-3

2. Reverse the Operations

a. Example 1. First consider a simple example **$f(x) = 3x + 2$**.The graph of f is a line with slope 3, so it passes the horizontal line test and does have an inverse.

b. There are two steps required to evaluate f at a number x. First we multiply x by 3, then we add 2.

c. Thinking of the inverse function as undoing what f did, we must undo these steps in reverse order.

d. The steps required to evaluate f^{-1} are to first undo the adding of 2 by subtracting 2. Then we undo multiplication by 3 by dividing by 3.

e. Therefore, $f^{-1}(x) = (x - 2)/3$.

3. **Algebraically – Steps for Funding the Inverse of a Function – *f*.**

a. Replace f(x) by y in the equation describing the function.

b. Interchange x and y. In other words, replace every x by a y and vice versa.

c. Solve for y.

d. Replace y by $f^{-1}(x)$

Example 2. **f(x) = 6 – (x/2)**

$y = 6 - \frac{x}{2}$

$x = 6 - \frac{y}{2}$

$x = -\frac{y}{2} + 6$

$-6 \qquad -6$

$(-2)\ x - 6 = -\frac{y}{2}\ (-2)$

$-2x + 12 = y$

$-2(x-6) = y$

$f^{-1}(x) = -2x + 12$

Example 3. **f(x) = x³ + 2**

$y = x^3 + 2$

$x = y^3 + 2$

$-2 \qquad -2$

$\sqrt[3]{x-2} = \sqrt[3]{y^3}$

$\sqrt[3]{x-2} = y$

$f^{-1}(x) = \sqrt[3]{x-2}$

Example 3. **f(x) = x³ + 2**