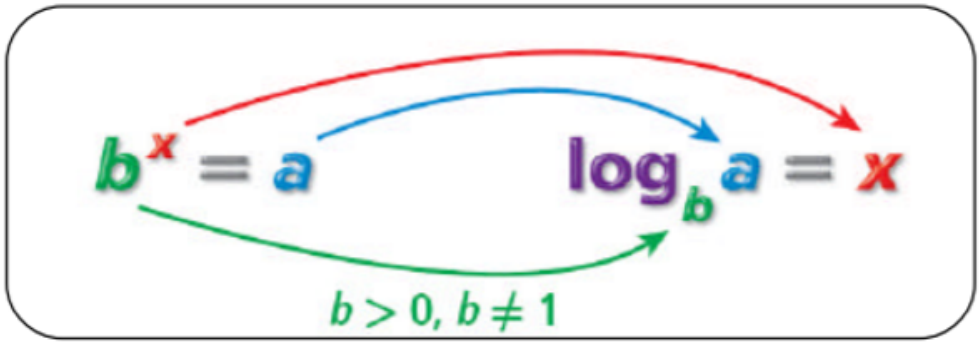
Introduction to Logarithms

Logs are the inverse of an exponential

You can convert any exponential to a log or a log to an exponential.

To convert you use the following



You read a log as follows $\log_b a = x$ "log ***base*** b ***of*** a ***is*** x"

There are 3 types of logs

Written	$y = \ln x$	$y = \log x$	$y = \log_b x$
Base	e	10	b

Example 1: Write each exponential form in logarithmic form.

equal
base = of

Exponential Form	Logarithmic Form
$3^5 = 243$	$\log_3 243 = 5$
$25 = 5^2$	$\log_5 25 = 2$
$10^4 = 10{,}000$	$\log 10{,}000 = 4$
$6^{-1} = x$	$\log_6 x = -1$
$a^b = c$	$\log_a c = b$

$\log_3 25$

$\log 25 = 2$

Example 2: Write each logarithmic form in exponential form.

Logarithmic Form	Exponential Form
$\log_9 9 = 1$	$9^1 = 9$
$\log_2 512 = 9$	$2^9 = 512$
$\log_8 2 = x$	$8^x = 2$
$\log_4 x = -2$	$4^{-2} = x$
$\log_b 1 = 0$	$b^0 = 1$

$9 = \log_2 512$

Special Properties of Logarithms

For any base b such that $b > 0$ and $b \neq 1$,

LOGARITHMIC FORM	EXPONENTIAL FORM	EXAMPLE
Logarithm of Base b		
$\log_b b = 1$	$b^1 = b$	$\log_{10} 10 = 1$ $10^1 = 10$
Logarithm of 1		
$\log_b 1 = 0$	$b^0 = 1$	$\log_{10} 1 = 0$ $10^0 = 1$

$\log_m 1 = 0 \quad \log_a a = 1$

$\log_{537} 1 = 0 \quad \log_3 3 = 1$

How to evaluate a log

1. set equal to x
2. Convert to an exponential
3. Solve the exponential

Evaluate each expression below.

1. $\log_3 243 = 5$
2. $\log_8 2 = \frac{1}{3}$
3. $\log_{27} 81 = \frac{4}{3}$
4. $\log_4 \frac{1}{32} = \frac{-5}{2}$

1. $\log_3 243 = x$
   $3^x = 243$
   $3^x = 3^5$
   $x = 5$

2. $\log_8 2 = x$
   $8^x = 2$
   $(2^3)^x = 2^1$
   $3x = 1$
   $x = \frac{1}{3}$

3. $\log_{27} 81 = x$
   $27^x = 81$
   $(3^3)^x = 3^4$
   $3x = 4$
   $x = \frac{4}{3}$

4. $\log_4 \frac{1}{32} = x$
   $4^x = \frac{1}{32}$
   $2^{2x} = 2^{-5}$
   $\frac{2x}{2} = \frac{-5}{2}$ $x = \frac{-5}{2}$