

1. Convert the following log to exponential form. $\log_2(x-1) = 3$

$$2^3 = x-1$$

2. Convert the following exponential to log form. $e^{x-1} = 5$

$$\ln 5 = x-1$$

$$\log_b a = c \Leftrightarrow b^c = a$$

3. Simplify the following without a calculator

$$\log_2 128 = x$$

$$2^x = 128$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

$$2^x = 2^7$$

$$x = 7$$

4. Simplify the following without a calculator

$$\log_{49} \left(\frac{1}{343} \right) = x$$

$$49^x = \frac{1}{343}$$

$$7^{2x} = 7^{-3}$$

$$\frac{2x}{2} = \frac{-3}{2}$$

$$x = -\frac{3}{2}$$

5. Simplify the following without a calculator

$$\log_{12} 1 = x$$

$$12^x = 1$$

$$12^x = 12^0$$

$$x = 0$$

6. Simplify the following without a calculator

$$\log \left(\frac{1}{1000} \right) = x$$

$$10^x = \frac{1}{1000} \quad \frac{1}{10^3}$$

$$10^x = 10^{-3}$$

$$x = -3$$

7. Use the change of base formula to rewrite the following log.

$$\log_x y$$

$$\frac{\log y}{\log x}$$

$$\frac{\ln y}{\ln x}$$

8. Use log properties to condense the following log.

$$2\log_4 x + 5\log_4 2 - 3\log_4 y$$

$$\log_4 x^2 + \log_4 2^5 - \log_4 y^3$$

$$\log_4 x^2 \cdot 2^5 - \log_4 y^3$$

$$\log_4 32x^2 - \log_4 y^3$$

$$\log_4 \frac{32x^2}{y^3}$$

9. Use log properties to condense the following log.

$$3\ln x - 2\ln y - 4\ln z$$

$$\ln x^3 - \ln y^2 - \ln z^4$$

$$\ln x^3 - (\ln y^2 + \ln z^4)$$

$$\ln \frac{x^3}{y^2 z^4}$$

10. Use log properties to expand the following log.

$$\log_2 \left(\frac{8x^3}{y^5} \right)$$

$$\log_2 8 + \log_2 x^3 - \log_2 y^5$$

$$\log_2 2^3 + \log_2 x^3 - \log_2 y^5$$

$$3\log_2 2 + 3\log_2 x - 5\log_2 y$$

$$3 + 3\log_2 x - 5\log_2 y$$

$$\ln \frac{x^3}{y^2 z^4}$$

$$3\log_x 2x^3 - \log_x (2x)^3 - 8x^3$$

11. Use the properties of logarithms and the values below to find the logarithm indicated.

Do not use a calculator to evaluate the logs.

$$\log_9 12 = R$$

$$\log_9 11 = S$$

$$\log_9 8 = T$$

Find $\log_9 121$

$$\begin{aligned} & \log_9 11 \cdot 11 \\ & \log_9 11 + \log_9 11 \\ & S + S \\ & \textcircled{2S} \quad \textcircled{2S} \end{aligned}$$

12. Use the properties of logarithms and the values below to find the logarithm indicated.

Do not use a calculator to evaluate the logs.

$$\log_8 12 = A$$

$$\log_8 19 = B$$

$$\log_8 9 = C$$

Find $\log_8 \left(\frac{64}{9} \right)$

$$\begin{aligned} & \log_8 64 - \log_8 9 \\ & \log_8 8^2 - C \\ & 2 \log_8 8 - C \\ & 2 - C \end{aligned}$$

Type 1: Log = a number

- 1. Convert to an exponential**
- 2. Solve the equation for the variable**
- 3. Check your solution**

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| <p>Example 1: $\log(4t) = 2$</p> $10^2 = 4t$ $\frac{100}{4} = \frac{4t}{4}$ $25 = t$ <p>Check $\log(4(25)) = 2$</p> | <p>Example 2: $\ln(x-1) = 3$</p> $e^3 = x-1$ $\frac{+1 \quad +1}{e^3 + 1 = x}$ $21.09 = x$ <p>Check $\ln(21.09-1) = 3$</p> | <p>Example 3: $\log_4 2 + \log_4(3+x) = 1$</p> $\log_4 2(3+x) = 1$ $\log_4(6+2x) = 1$ $4^1 = 6+2x$ $\frac{-6 \quad -6}{-2 = 2x}$ $\frac{-2}{2} = \frac{2x}{2}$ $x = -1$ <p>$\log_4 2 + \log_4(3-1) = 1$</p> |
| | | $\log 2 \div \log 4 + \log(3-1) \div \log 4$ |

Type 2: Log = Log

1. Cross out the log on both sides
2. Solve the equation for the variable
3. Check the solution

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| <p>Example 4: $\log_3(4t) = \log_3(3t-6)$</p> $\begin{array}{r} 4t = 3t - 6 \\ -3t \quad -3t \\ \hline t = -6 \end{array}$ <p style="text-align: center;">No Sol.</p> <p>Check $\log_3(4 \cdot -6) = \log_3(3 \cdot -6 - 6)$</p> | <p>Example 5: $\log_2(-x) = \log_2(2-3x)$</p> $\begin{array}{r} -x = 2 - 3x \\ +3x \quad +3x \\ \hline 2x = 2 \\ x = 1 \end{array}$ <p style="text-align: center;">No Sol.</p> <p>Check $\log_2(-1) = \log_2(2-3 \cdot 1)$</p> | <p>Example 6: $\log(x+5) = \log(2x+1)$</p> $\begin{array}{r} x+5 = 2x+1 \\ -x \quad -x \\ \hline 5 = x+1 \\ -1 \quad -1 \\ \hline 4 = x \end{array}$ <p>Check $\log(4+5) = \log(2 \cdot 4 + 1)$ $.95 = .95$</p> |
| $\log(4 \cdot -6) \div \log 3$ | $\log(-1) \div \log(2)$ | |

Type 3 Exponential = Number**1. Convert to a log****2. Solve for the variable**

$$b^{ex} = \text{Something}$$

Example 7:

$$8^x = 6$$

$$\log_8 6 = x$$

$$\frac{\log 6}{\log 8} = x$$

$$.86 = x$$

Example 8:

$$4 = 15 - e^{x-8}$$

$$\begin{array}{r} -15 \\ -15 \\ \hline -11 = -e^{x-8} \end{array}$$

$$11 = e^{x-8}$$

$$\ln(11) = x - 8$$

$$\ln(11) + 8 = x$$

$$10.40 = x$$

Example 9:

$$e^x + 33 = 12e^x$$

$$\begin{array}{r} -e^x \\ -e^x \\ \hline 33 = 11e^x \end{array}$$

$$3 = e^x$$

$$\ln 3 = x$$

$$1.099 = x$$

$$\ln(11) = x - 8$$

$$\begin{array}{r} 2.398 = x - 8 \\ +8 \quad +8 \end{array}$$

Type 4 Exponential = Exponential**1. Get the bases the same****2. Cross out the bases****3. Solve for the variable**

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| Example 10: $2^{x-2} = 2^3$ $\begin{array}{r} x-2 = 3 \\ +2 \quad 2 \\ \hline x = 5 \end{array}$ | Example 11: $8^x = 2$ $\begin{array}{r} 2^{3x} = 2^1 \\ 3x = 1 \\ x = \frac{1}{3} \end{array}$ |
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